Department of Communications Engineering

Communication Systems

Third Year Class

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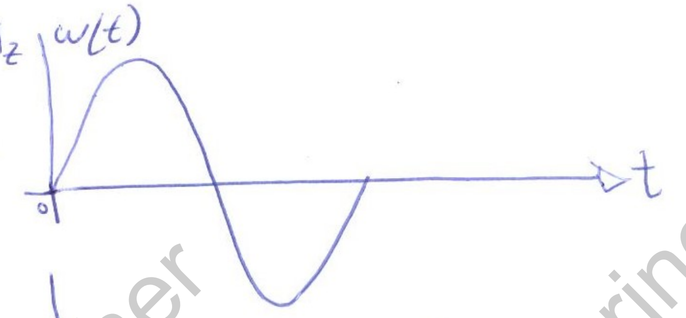
Lecture 17

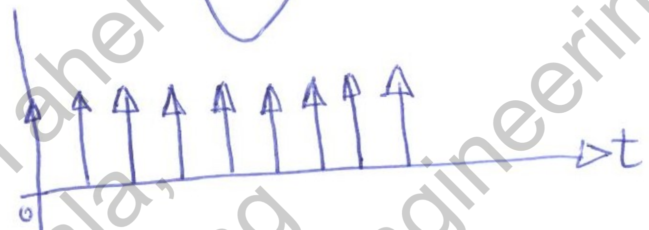
Pulse Modulation, Pulse Amplitude Modulation (PAM) II

Flat-Top Sampling PAM :

- $w(t)$ is an analog bandlimited to $B\,Hz$
- The Flat-Top PAM signal will be

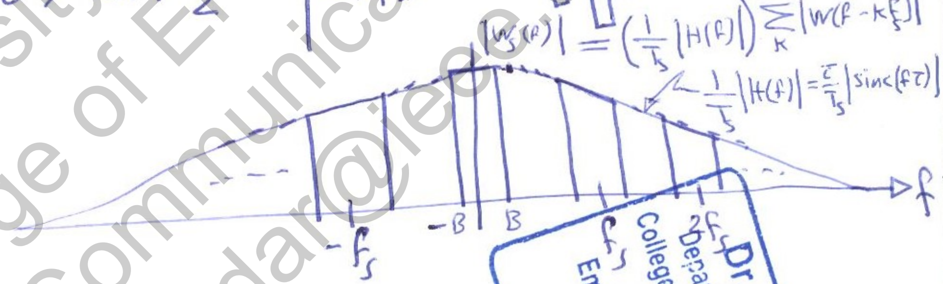


$$w_s(t) = \sum_{k=-\infty}^{\infty} w(kT_s)\, h(t - kT_s)$$



$h(t)$ is the shape of the sampling waveform,

$$h(t) = rect\left(\frac{t}{\tau}\right) = \Pi\left(\frac{t}{\tau}\right) = \begin{cases} 1, & |t| < \frac{\tau}{2} \\ 0, & |t| > \frac{\tau}{2} \end{cases}$$



where $\tau \leq T_s = \frac{1}{F_s}$

$F_s \geq 2B$

Hence : $W_s(f) = \frac{1}{T_s} H(f) \sum_{k=-\infty}^{\infty} W(f - kf_s)$

where $H(f) = \tau\, \mathrm{sinc}(\tau f)$

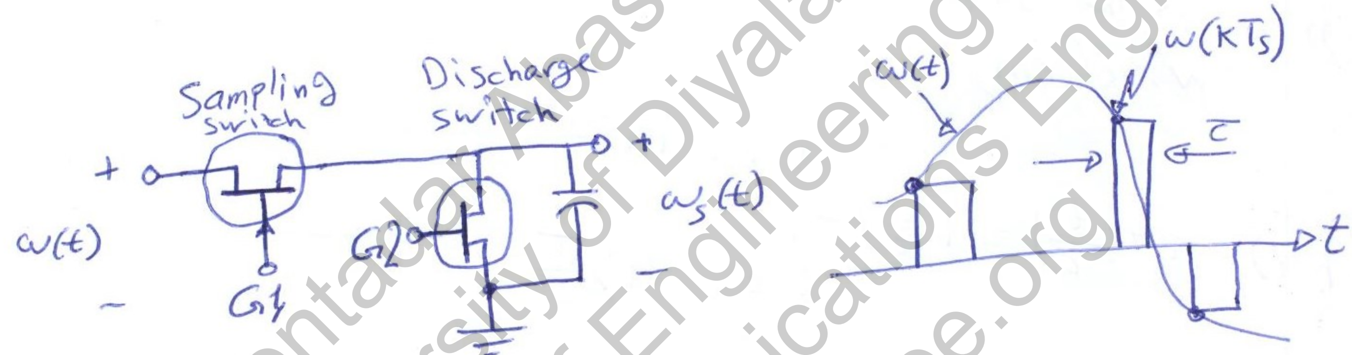
$$\therefore W_s(f) = \frac{\tau}{T_s}\, \mathrm{sinc}(\tau f) \sum_{k=-\infty}^{\infty} W(f - kf_s)$$

$$W_s(f) = \tau F_s\, \mathrm{sinc}(\tau F) \sum_{k=-\infty}^{\infty} W(f - kf_s)$$

* Flat-Top PAM consists of instantaneous samples at $t = kT_s$,

* The sample values are determined by $w(kT_s)$

* Flat-top PAM can be generated using a circuit called sample-and-Hold (S/H).



* To recover the signal:- LPF can be used.

* using LPF to demodulate the PAM signal will cause losses in the high frequency because of the LPF $H(f)$.

* To reduce the effect of $H(f)$, τ can be reduced, or by using an equalization filter of transfer function $\frac{1}{H(f)}$.

* Another method to recover $w(t)$ from $w_s(t)$ (demodulation) is by using the product detection.

* Because of a rectangular basis of the PAM, the bandwidth is very large.

* PAM can not be used for long distances.

Q/ consider PAM transmission of a voice signal with $W \cong 3\,KHz$.
calculate total bandwidth if $F_s = 8\,KHz$ and $\tau = 0.1 T_s$.

solution Given $W = 3\,KHz$, $F_s = 8\,KHz$, $\tau = 0.1\,T_s$ sec.

we know $\tau \leq T_s$

and we know $T_s = \frac{1}{F_s}$

$$F_s = 2B$$

$$\therefore T_s = \frac{1}{2B}$$

$$\therefore \tau \leq \frac{1}{2B}$$

$$\therefore \boxed{B = \frac{1}{2\tau}}$$ general rule for PAM (all types)

since $\tau = 0.1\,T_s$

$$\tau = \frac{0.1}{F_s}$$

$$\therefore B = \frac{1}{2\frac{0.1}{F_s}} = \frac{F_s}{0.2}$$

$$\therefore B = \frac{F_s}{0.2} = \frac{8000}{0.2}$$

$$\boxed{B = 40\,KHz}$$

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Q/ pulse-amplitude modulated waveform $g(t) = \sin(2\pi \frac{1}{T_m} t)$, where $T_m = 0.2$ seconds, sampled at $f_s = 100$ Hz, calculate the total transmission bandwidth if the duty cycle was $0.25 T_s$.

solution Given $\tau = 0.25 T_s$ seconds

$F_s = 100$ Hz

$$B \leqslant \frac{1}{2\tau} = \frac{1}{2 \times 0.25 T_s} = \frac{F_s}{0.5} = \frac{100}{0.5} = 200 \text{ Hz}.$$

Q/ A rectangular PAM signal $g(t) = 2\,\text{rect}(t)$, sampling frequency was $F_s = 150$ Hz. The sampling duty cycle was 10% of the sampling period, what is the transmission bandwidth?

solution

Given $F_s = 150$ Hz, $\tau = 0.1 T_s$

$$B \leqslant \frac{1}{2\tau} = \frac{1}{2 \times 0.1 T_s} = \frac{F_s}{0.2} = \frac{150}{0.2}$$

$B = 750$ Hz.

Equalization of PAM

Suppose the analog signal is $w(t)$, if sampled at kT_s instances, then

$$w_p(t) = \sum_k w(kT_s)\, p(t - kT_s) \qquad (1)$$

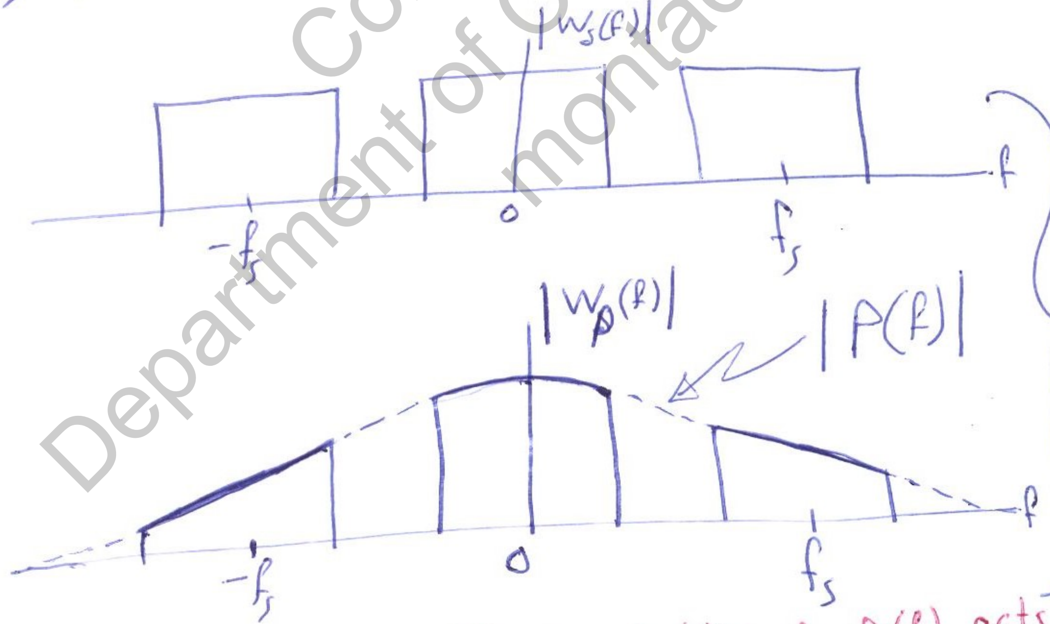
where $p(t-kT_s) = p(t) * \delta(t - kT_s)$

$$\therefore\; w_p(t) = p(t) * \left[\sum_k w(kT_s)\,\delta(t - kT_s)\right] = p(t) * w_s(t)$$

In Frequency-Domain:-

$$W_p(f) = P(f)\left[f_s \sum_n W(f - nf_s)\right] = P(f)\, W_s(f) \qquad (2)$$

Lets consider the input signal $W(f) = \mathrm{rect}\left(\frac{f}{2W}\right)$



Just like passing an ideal sampled wave through filter of transfer function $P(f)$.

The high-frequency rolloff characteristic of $P(f)$ acts like a LPF and attenuates the upper portion of the message signal's spectrum, which is called aperture effect.

* The Larger the pulse duration or aperture τ, the larger the effect.

* Aperture effect can be corrected in reconstruction by including an equalizer with

$$\boxed{H_{eq}(f) = \frac{K e^{-j\omega t_d}}{P(f)}} \quad \text{—— (3)}$$

Equalization is needed only when $\frac{\tau}{T_s} \ll 1$

Unipolar Flat-Top PAM

unipolar flat-top PAM is defined as

$$\boxed{w_p(t) = \sum_K A_o \left[1 + \mu\, w(KT_s)\right] p(t - KT_s)} \quad \text{—— (4)}$$

where A_o is the unmodulated pulse-amplitude,
μ is the modulation index (controls the amount of amplitude variation).

To ensure unipolar signal (single-polarity) the following condition must satisfied :

$$\boxed{1 + \mu\, w(t) > 0} \quad \text{—— (5)}$$

Hence the bandwidth is $B \geq \frac{1}{2\tau} \gg W$ —— (6)
W is the bandwidth of the message.

Q1/ Sketch $|X_p(f)|$ and find $H_{eq}(f)$ for flat-top sampling with $\tau = \frac{T_s}{2}$, $F_s = 2.5\,W$, where W is the bandwidth of the message $x(t)$, and $p(t) = \text{rect}\left(\frac{t}{\tau}\right)$.
Is equalization essential in this case?

Solution

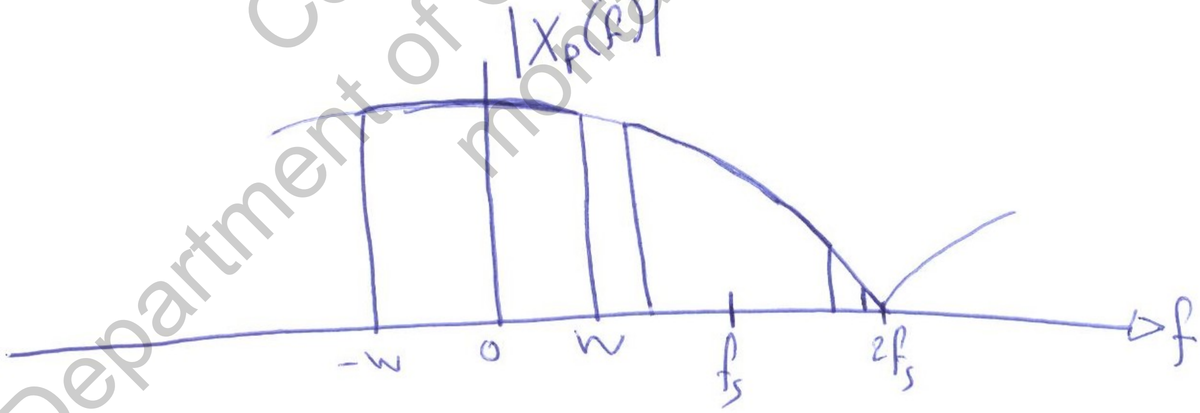
$$p(t) = \text{rect}\left(\frac{t}{\tau}\right) \xleftrightarrow{F.T.} P(f) = \tau \,\text{sinc}(f\tau)$$

$$\therefore P(f) = \tau \,\text{sinc}(f\tau) = \frac{T_s}{2}\,\text{sinc}\left(\frac{f}{2f_s}\right)$$

$$H_{eq}(f) = \frac{K}{\text{sinc}\left(\frac{f}{2f_s}\right)} = \frac{K}{\text{sinc}\left(\frac{f}{5W}\right)} \quad \text{for } |f| \leq W$$

$$H_{eq}(0) = K, \quad H_{eq}(W) = \frac{K}{\text{sinc}\left(\frac{W}{5W}\right)} = \frac{K}{\text{sinc}(0.2)} = 1.07K$$

So, equalization is not essential.



Q2/ Sketch $|X_p(f)|$ and find $H_{eq}(f)$ for flat-top sampling with $\tau = T_s/2$, $F_s = 2.5W$, and $p(t) = \cos(\pi t/\tau)\,\text{rect}(\frac{t}{\tau})$, Is equalization essential in this case?

Solution

$$p(t) = \cos\left(\frac{\pi t}{\tau}\right)\text{rect}\left(\frac{t}{\tau}\right)$$

$$P(f) = \frac{\tau}{2}\left[\text{sinc}(f\tau - \tfrac{1}{2}) + \text{sinc}(f\tau + \tfrac{1}{2})\right]$$

$$= \frac{T_s}{4}\left[\text{sinc}\left(\frac{f - f_s}{2f_s}\right) + \text{sinc}\left(\frac{f + f_s}{2f_s}\right)\right]$$

$$H_{eq}(f) = K\left[\text{sinc}\left(\frac{f - 2.5W}{5W}\right) + \text{sinc}\left(\frac{f + 2.5W}{5W}\right)\right]^{-1}$$

$H_{eq}(0) = 0.785K$, $H_{eq}(W) = 0.816K$

So, equalization is not essential.